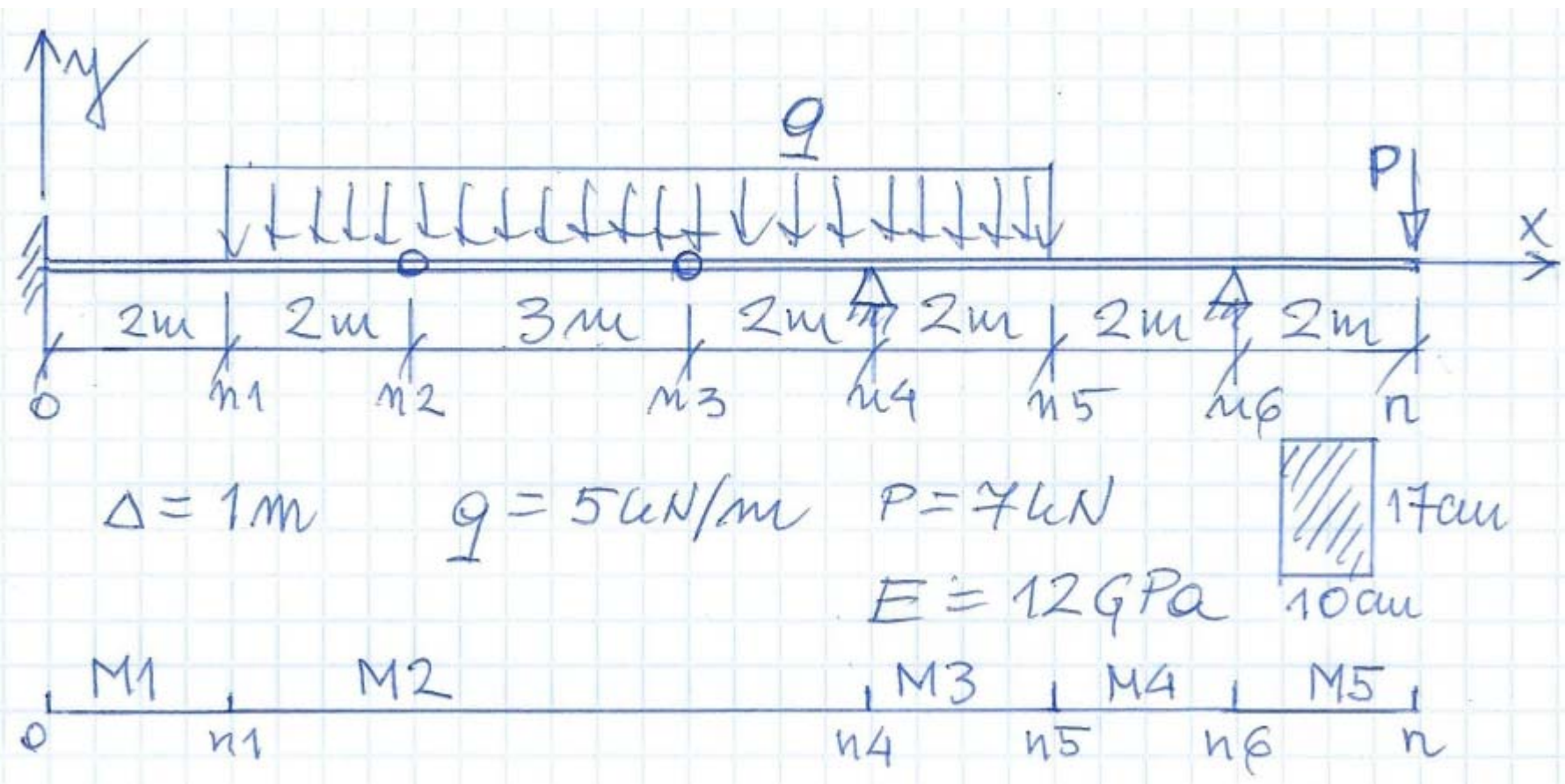


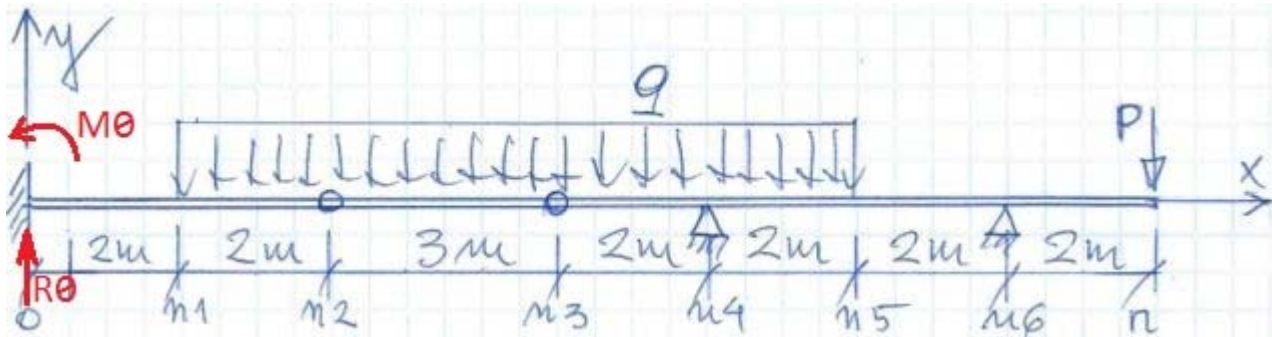
# Obliczenie ugięcia belki metodą różnic skończonych (MRS)

ORIGIN := 0



$$b := 10cm \quad h := 17cm \quad \underline{J} := \frac{b \cdot h^3}{12} \quad E := 12GPa$$

$$q := 5 \frac{kN}{m} \quad P := 7kN \quad \Delta := 1m \quad \alpha := \frac{\Delta^2}{E \cdot J} = 2.035 \cdot \frac{1}{MN}$$



$$L1 := 2m \quad L2 := 4m \quad L3 := 7m \quad L4 := 9m \quad L5 := 11m \quad L6 := 13m \quad L := 15m$$

$$n1 := \frac{L1}{\Delta} \quad n2 := \frac{L2}{\Delta} \quad n3 := \frac{L3}{\Delta} \quad n4 := \frac{L4}{\Delta} \quad n5 := \frac{L5}{\Delta} \quad n6 := \frac{L6}{\Delta} \quad n := \frac{L}{\Delta} \quad n = 15$$

$$T2 := \frac{q \cdot 3m \cdot 1.5m}{3m} = 7.500 \cdot kN \quad T3 := T2 = 7.500 \cdot kN$$

$$M0 := T2 \cdot 4m + q \cdot 2m \cdot 3m = 60.000 \cdot kN \cdot m \quad R0 := q \cdot 2m + T2 = 17.500 \cdot kN$$

$$R4 := \frac{T3 \cdot 6m + q \cdot 4m \cdot 4m - P \cdot 2m}{4m} = 27.750 \cdot kN \quad R6 := T3 + q \cdot 4m + P - R4 = 6.750 \cdot kN$$

$$M_1(x) := R_0 \cdot x - M_0$$

$$M_2(x) := M_1(x) - q \cdot \frac{(x - L_1)^2}{2}$$

$$M_3(x) := M_2(x) + R_4 \cdot (x - L_4)$$

$$M_5(x) := -P \cdot (L - x)$$

$$M_4(x) := M_5(x) + R_6 \cdot (L_6 - x)$$

$$i := 0 \dots n \quad X_i := i \cdot \Delta$$

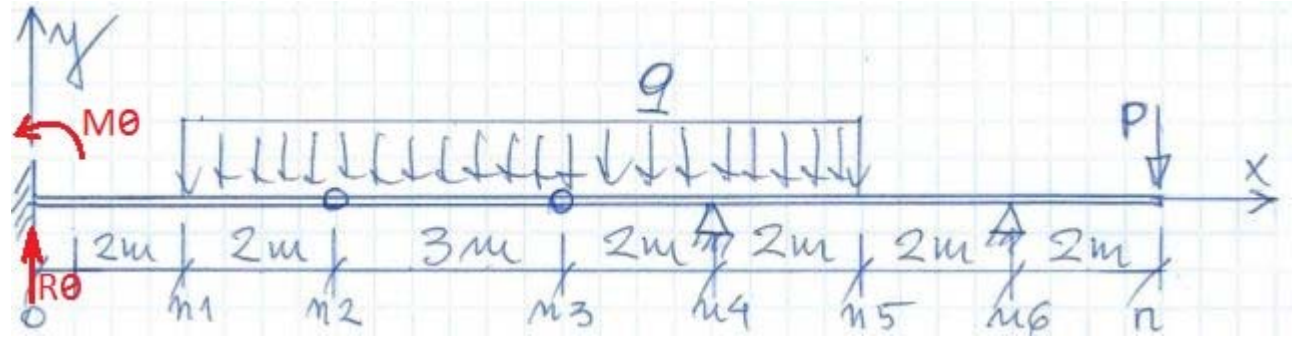
$$i := 0 \dots n_1 \quad M_i := M_1(X_i)$$

$$i := n_1 + 1 \dots n_4 \quad M_i := M_2(X_i)$$

$$i := n_4 + 1 \dots n_5 \quad M_i := M_3(X_i)$$

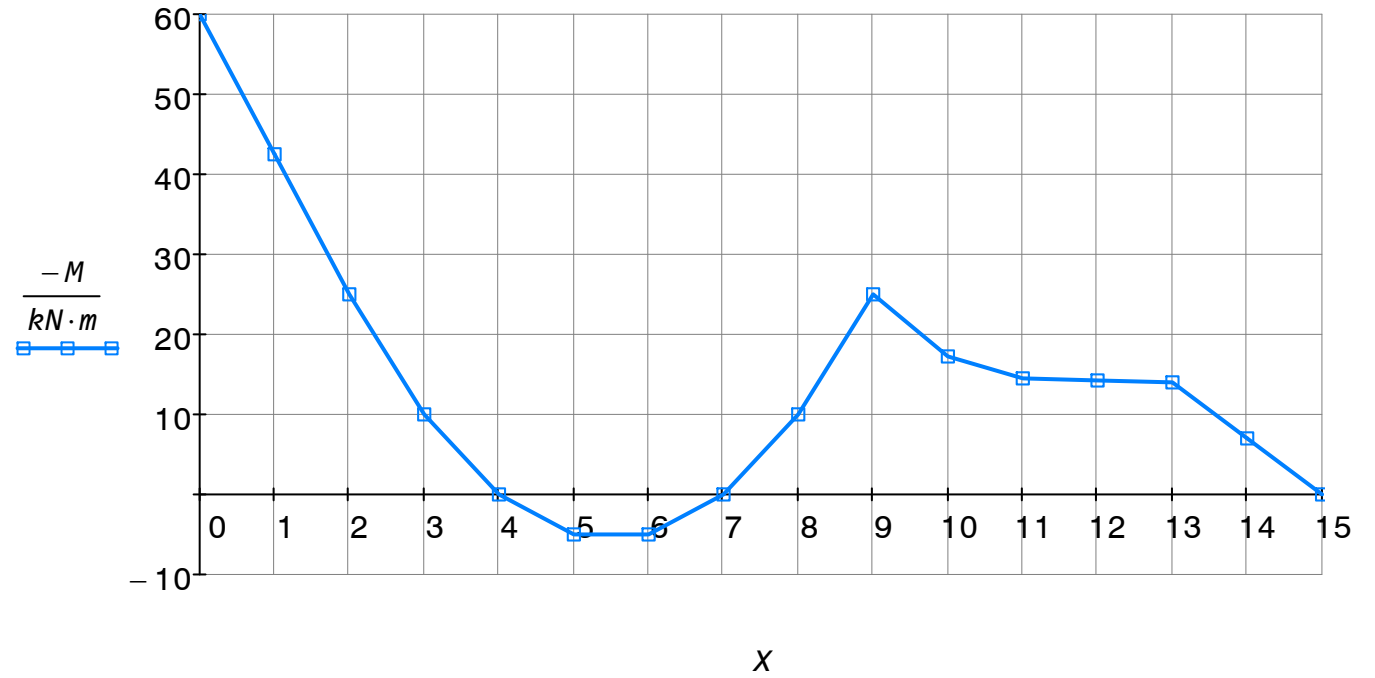
$$i := n_5 + 1 \dots n_6 \quad M_i := M_4(X_i)$$

$$i := n_6 + 1 \dots n \quad M_i := M_5(X_i)$$



## Wykres momentów zginających

$X =$	$0$	$\cdot m$	$M =$	$0$	$\cdot kN \cdot m$
0	0.0		0	-60.00	
1	1.0		1	-42.50	
2	2.0		2	-25.00	
3	3.0		3	-10.00	
4	4.0		4	0.00	
5	5.0		5	5.00	
6	6.0		6	5.00	
7	7.0		7	0.00	
8	8.0		8	-10.00	
9	9.0		9	-25.00	
10	10.0		10	-17.25	
11	11.0		11	-14.50	
12	12.0		12	-14.25	
13	13.0		13	-14.00	
14	14.0		14	-7.00	
15	15.0		15	0.00	



$$\max(M) = 5.000 \cdot kN \cdot m$$

$$\min(M) = -60.000 \cdot kN \cdot m$$

## Układ równań MRS:

$$i := 0 .. n \quad A_{i, i} := -2$$

$$i := 1 .. n \quad A_{i, i-1} := 1$$

$$i := 0 .. n-1 \quad A_{i, i+1} := 1$$

$A =$

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0

Warunki brzegowe:  $y_0 = 0$   $\varphi_0 = 0$   $\dashrightarrow 2y_1 = \alpha M_0$   
 $w=n2, k=0$   $w=0, k=1$

$j := 0 .. n$   $y_{n4} = 0$   $y_{n6} = 0$   
 $w=n3, k=n4$   $w=n, k=n6$

$w := n2$   $k := 0$   $A_{w,j} := 0$   $A_{w,k} := 1$

$w := 0$   $k := 1$   $A_{w,j} := 0$   $A_{w,k} := 2$

$w := n3$   $k := n4$   $A_{w,j} := 0$   $A_{w,k} := 1$

$w := n$   $k := n6$   $A_{w,j} := 0$   $A_{w,k} := 1$

$A =$

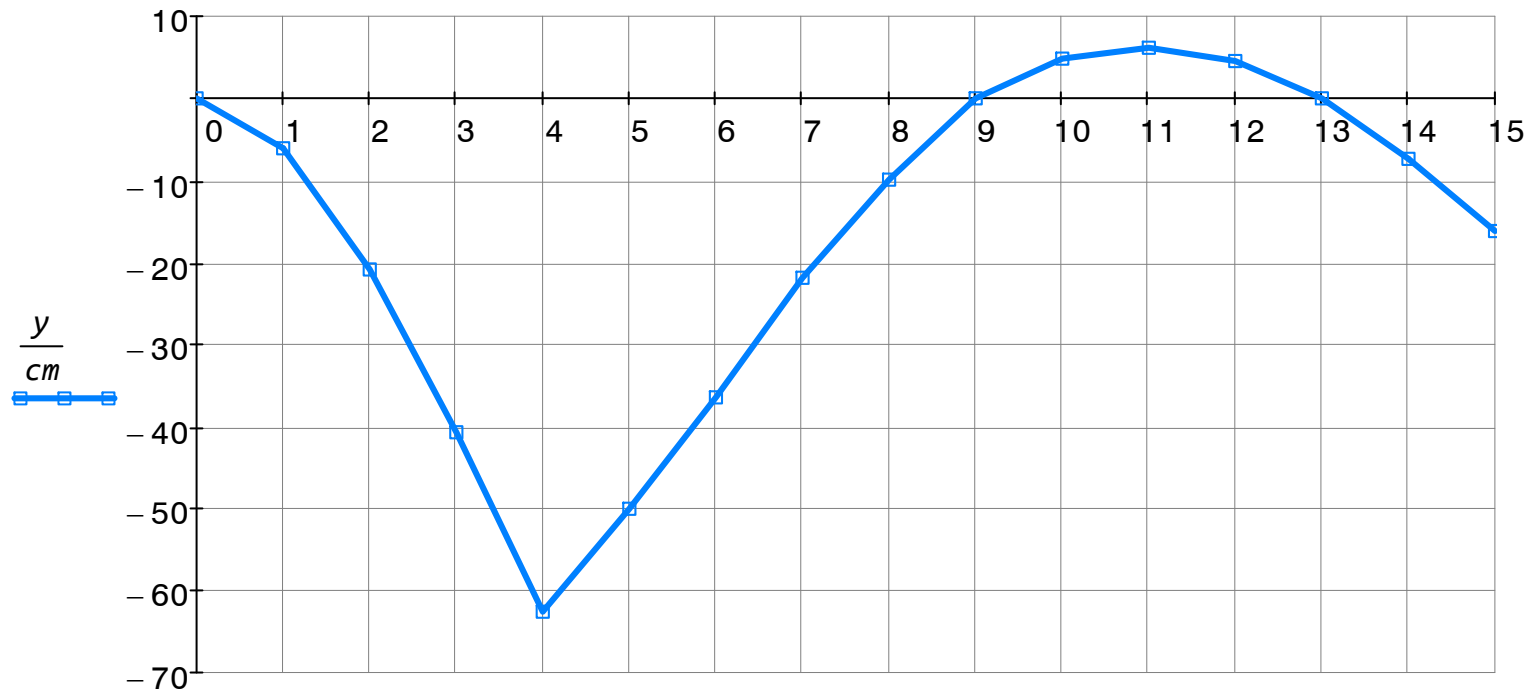
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0.0	2.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	-2.0	1.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0

$|A| = -24.000$

$$y := \text{lsolve}(A, \alpha \cdot M)$$

### Wykres ugięć belki

	0
0	-0.000
1	-6.106
2	-20.863
3	-40.708
4	-62.589
5	-50.037
6	-36.468
7	-21.881
8	-9.923
9	0.000
10	4.834
11	6.157
12	4.529
13	0.000
14	-7.378
15	-16.182



$$\min(y) = -62.589 \cdot \text{cm}$$

$$\max(y) = 6.157 \cdot \text{cm}$$