INFLUENCE OF THE LIMIT STATE CRITERION ON DIRECTION OF THE CRACK PROPAGATION IN THE ELASTIC-BRITTLE MATERIAL

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1. Introduction

The problem of crack initiation and propagation in the brittle materials, considering its practical significance, has been analysed by many researchers for the last two decades. Both, analytical and numerical method, e.g. Finite Elements Method (FEM) and Boundary Element Method, were used to solve this problem.

In the FEM approach, the mesh shape and density are very significant to convergence of calculated results with observed test results. In the advanced FEM analysis the “remeshing” technique and prediction of the crack propagation direction are used. This technique was used in our earlier work [2,3] but it provides some complications in FEM solving procedures.

In some special kind of problems, like tension test or rock cut test, another, simple procedure can be used to calculate path of the crack and forces causing material failure. This procedure is called “dead elements” in some FEM implementations and consist in removing (one or more) elements or changing element stiffness after checking the failure criterion for each element.

The shortcoming of this procedure is necessity to use more dense element mesh than in the “remeshing” procedures.

In presented paper “dead elements” procedure is used to analyse influence of the failure criterion on shape and direction of the crack and critical forces causing crack propagation.

2. Limit state conditions

The three failure criteria (Fig. 1) had been considered to analysis:

- author’s (*PJ)* criterion, proposed in 1986 [1], which limit state depends on three tensor invariants (I_1, J_2, J_3)
- well known Drucker-Prager criterion, (I_1, J_2)
- classical Huber-Mises criterion (J_2)

Limit curves described by eqs. (1), (2), (3) in biaxial stress state are shown in Fig. 1. Fig. 2 shows “tension meridian” and “compression meridian” of the ***PJ*** and Drucker-Prager limit surface in $\tau_0 - \sigma_0$ plane and Fig. 3 shows isometric view of this surfaces.

2.1 PJ criterion

The ***PJ*** criterion was proposed by the author in 1986 [1] in the form:

$$s_0 - C_0 + C_1 P(J) t_0 + C_2 t_0^2 = 0, \qquad (1)$$

where:

$P(J) = \cos\left(\frac{1}{3}\arccos(a\,J) - b\right)$ - function describing the shape of limit surface in deviatoric plane,

$s_0 = \frac{1}{3} I_1$	- mean stress,
$t_0 = \sqrt{\frac{2}{3} J_2}$	- octahedral shear stress,
I_1	- first invariant of the stress tensor,
$J_2,\ J_3$	- second and third invariant of the stress deviator,
$J = \dfrac{3\sqrt{3}\,J_3}{2\,J_2^{3/2}}$	- alternative invariant of the stress deviator,
$a,\ b,\ C_0, C_1, C_2$	- material constants.

Classical failure criteria, like Huber-Mises, Tresca, Drucker-Prager, Coulomb-Mohr as well as some new ones proposed by Lade, Matsuoka Ottosen, are particular cases [cf. 1,2] of the general form (1) ***PJ*** criterion.

Material constants can be evaluated on the basis of some simple material test results like:

- § f_c - failure stress in uniaxial compression,
- § f_t - failure stress in uniaxial tension,
- § f_{cc} - failure stress in biaxial compression at $\sigma_1/\sigma_2 = 1$,
- § f_{0c} - failure stress in biaxial compression at $\sigma_1/\sigma_2 = 2$,
- § f_v - failure stress in triaxial tension at $\sigma_1/\sigma_2/\sigma_3 = 1/1/1$,

For concrete or rock-like materials some simplifications can be taken on the basis of test results in biaxial stress state and R. M. Haythornthwaite "tension cutoff" hypotesis:

$$f_{cc}=1.1\, f_c\,, \qquad f_{0c}=1.25\, f_c\,, \qquad f_v= f_t.$$

2.2 Drucker – Prager criterion

With notation used in eq. (1) well-known Drucker–Prager criterion can be written:

$$s_0 - C_0 + C_1 t_0 = 0\,. \tag{2}$$

Two material constants C_0 and C_1 can be evaluated on the basis of uniaxial test results like f_t and f_c .

2.3 Huber – Mises criterion

Classical criterion proposed by T. Huber and R. von Mises can be obtained by simplification of the general form (1):

$$t_0 - C_0 = 0\,. \tag{3}$$

Material constant C_0, in this analysis, is evaluated with uniaxial tension failure stress f_t.

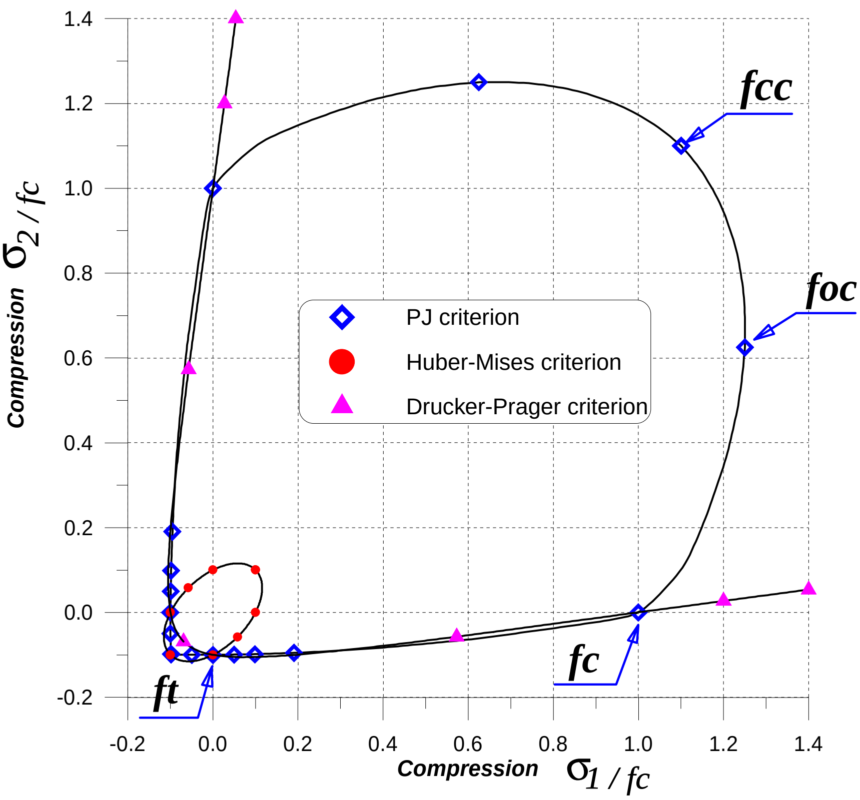


Fig. 1. Limit curves in biaxial state of stress

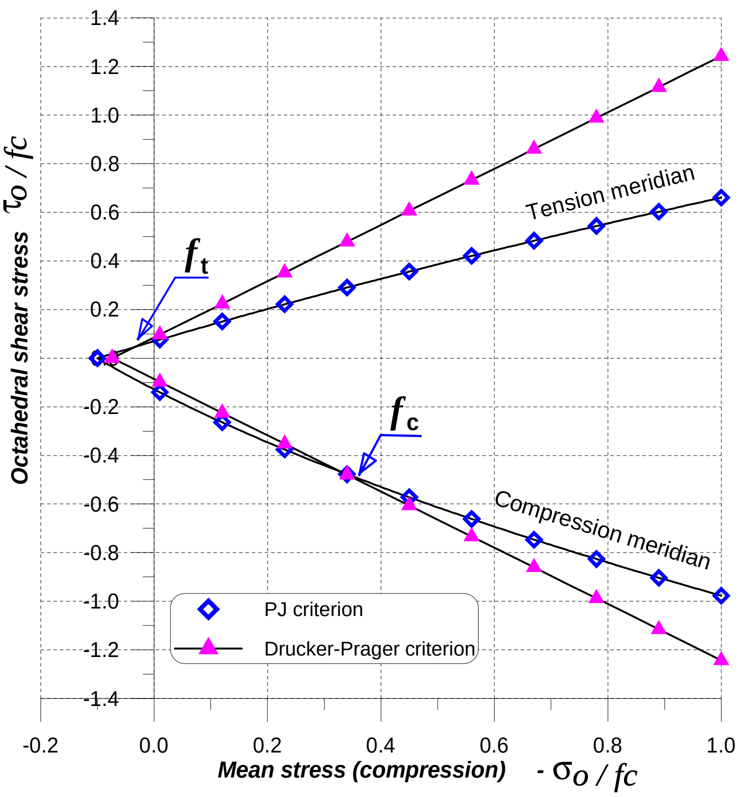


Fig. 2. ***PJ*** and Drucker-Prager limit surface cross section by $\tau_0 - \sigma_0$ plane.

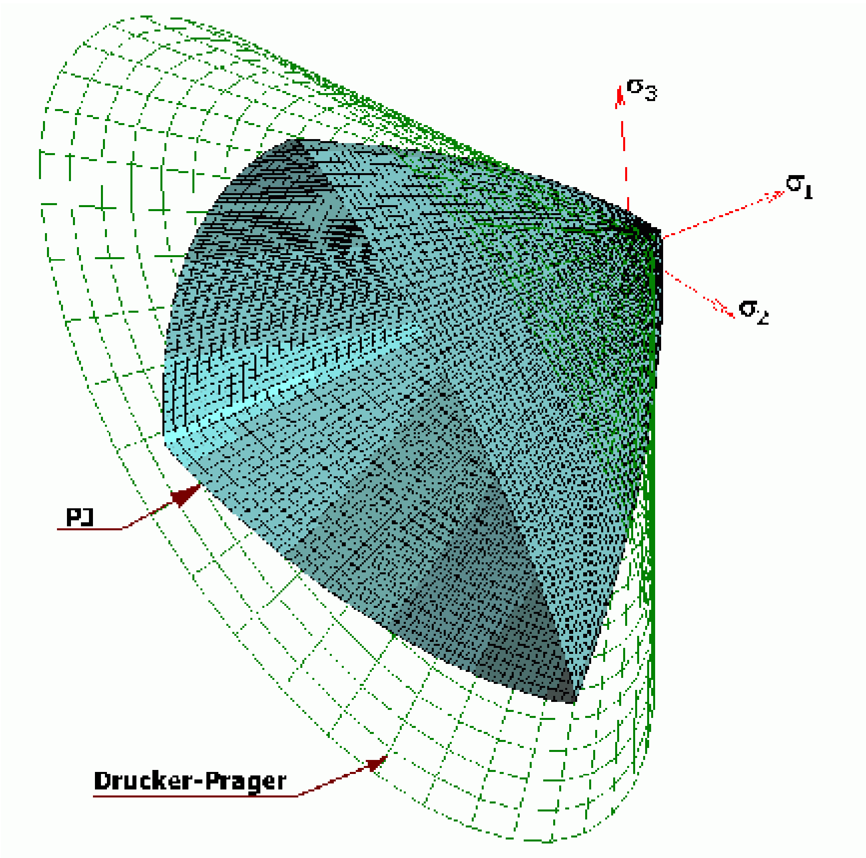


Fig. 3. ***PJ*** and Drucker-Prager limit surface – isometric view.

3. Finite element models and analysis method

The geometric parameters of the two models, which were analysed, are shown on Fig. 4 and Fig. 5. Model *"A"* modelled the test with tension stress domination and model *"B"* modelled rock cutting process in which compression stress is dominating.

Boundary conditions for model *"A"*:

- $z = -1 \rightarrow u_z=0,\ s_{zy}=0,$
- $y = 0 \rightarrow u_y=0,\ s_{yz}=0,$
- $y = 1 \rightarrow s_y=p.\ s_{yz}=0,$
- $y=-2 \rightarrow s_y=0,\ s_{yz}=0.$

Boundary conditions for model *"B"*:

- $z=-1 \rightarrow u_z=0,\quad s_{zy}=0,$
- $y=-2 \rightarrow u_y=0,\quad u_z=0,$
- $y=0 \rightarrow s_y=-p,\quad s_{yz}=0,$
- $y=1 \rightarrow s_y=0,\quad s_{yz}=0.$

Material constants for concrete or rock-like material were taken as follows:

- strength in uniaxial compression f_c=20MPa,
- strength in biaxial compression f_{cc}=22MPa, f_{0c}=25MPa,
- strength in uniaxial tension f_t=2MPa.
- Young modulus: E=32.4GPa, Poisson ratio: v=0,167.

Following step-by-step procedure for crack propagation has been executed during finite element analysis:

1. stress calculation for initial value of the force *P*,
2. search for the element with maximal value of critical stress correspond to considered failure criterion,
3. evaluation of the *P=Pcr* force for which the element with maximum stress is in the critical state according to failure criterion,
4. removing the chosen element from the analysed FEM mesh or changing its stiffness,
5. start next step of crack propagation process.

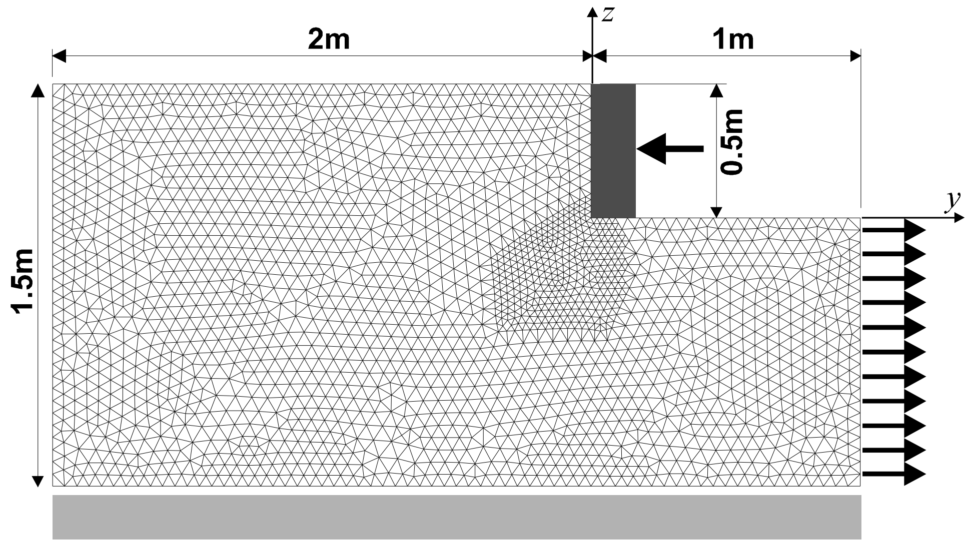


Fig. 4. Geometric parameters of the model *"A"*. Mesh with 2079 nodes.

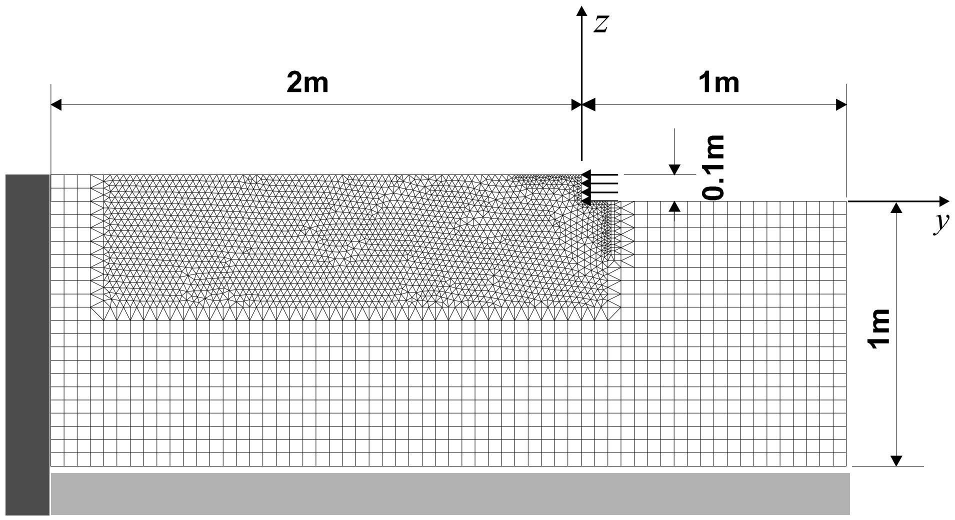


Fig. 5. Geometric parameters of the model *"B"*. Mesh with 3002 nodes.

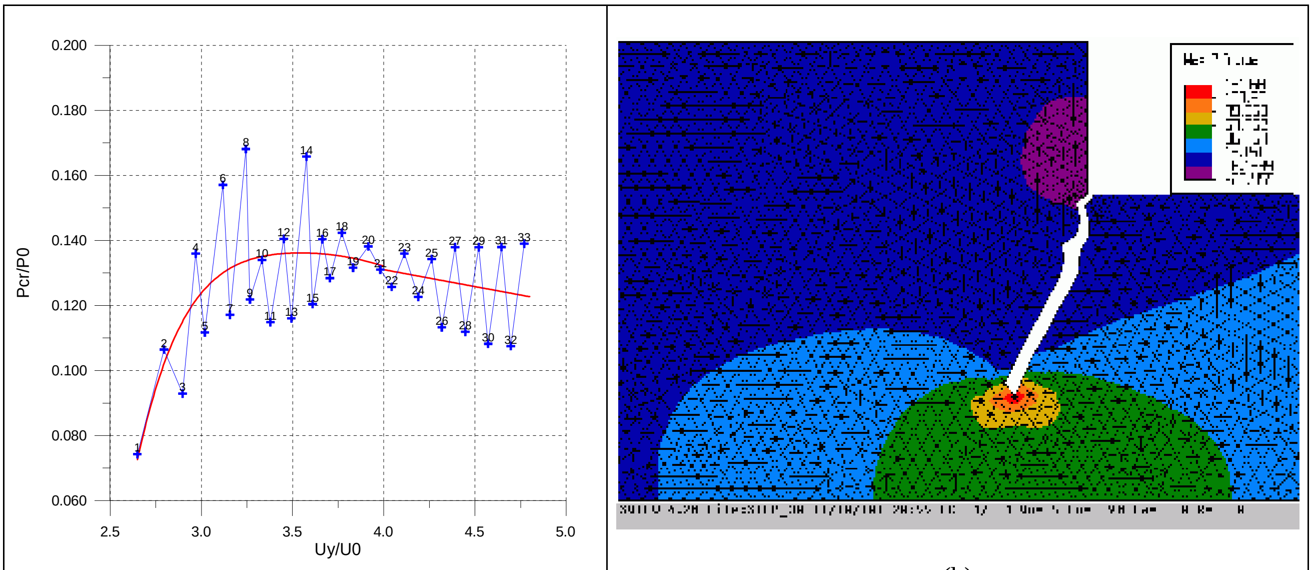
Calculations were done with Algor FEA software and author's additional module for failure criterion checking and mesh or element stiffness modifying.

4. Crack propagation analysis

From many cases of crack propagation process analyzed only a six cases will be shown in this paper. These cases are different in failure criteria or FEM meshes analyzed:

1. ***PJ*** criterion, model *"A"*, mesh with 1844 nodes,
2. ***PJ*** criterion, model *"A"*, mesh with 2079 nodes,
3. Drucker-Prager criterion, model *"A"*, mesh with 2079 nodes,
4. Huber-Mises criterion, model *"A"*, mesh with 2079 nodes,
5. ***PJ*** criterion, model *"B"*, mesh with 3002 nodes,
6. Drucker-Prager criterion, model *"B"*, mesh with 3002 nodes,

Results of calculations for chosen cases are shown in figures below. Each of them includes two parts: a) - P_{cr}/P_0 - U_y/U_0 chart, and b) – stress map with crack path. Broken lines with crosses on the charts represent values of the critical force P_{cr} causing crack propagation on each step of FEM analysis, numbers printed near the crosses are the step numbers. Smooth curves printed on the charts are fit lines for calculated values of P_{cr}. On the horizontal axis, U_y is the horizontal displacement of the monitoring node (y=0, z=-0.5). Force P_0 and displacement U_0 are conventional values taken as follows: $P_0 = f_t \cdot 1\mathrm{m}^2$, $U_0 = 1\mathrm{m} \cdot f_t / E$.



(a) (b)

Fig. 6. Case #1 - ***PJ*** criterion, model *"A"*, mesh with 1844 nodes.

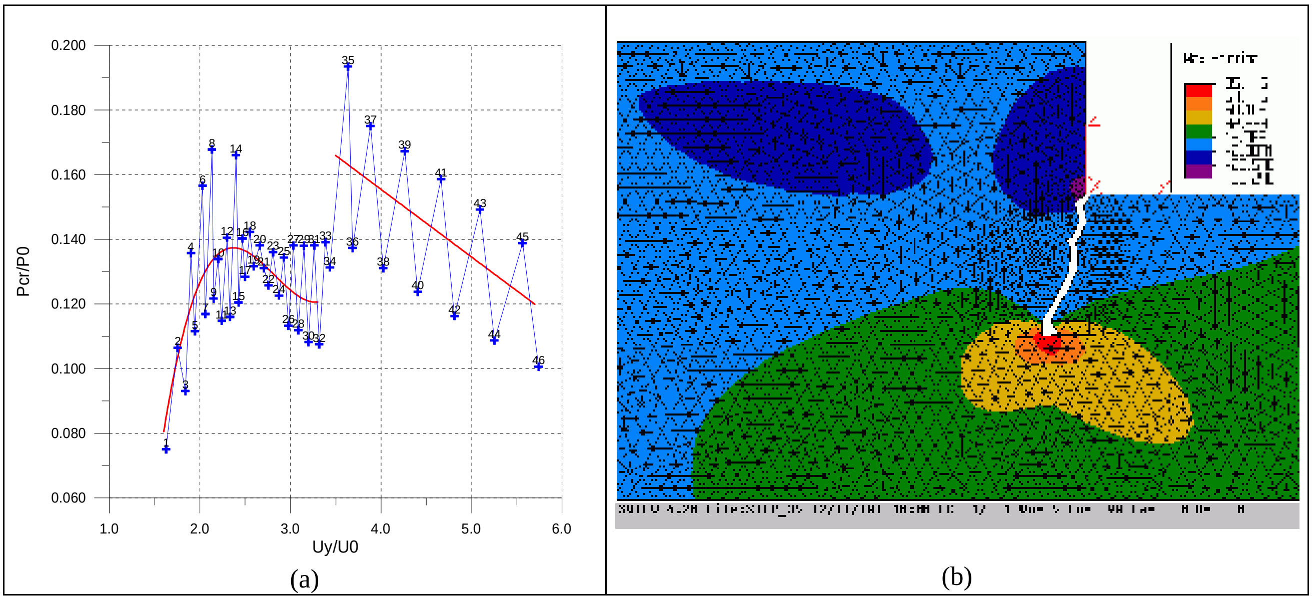


Fig. 7. Case #2 - ***PJ*** criterion, model *“A”*, mesh with 2079 nodes.

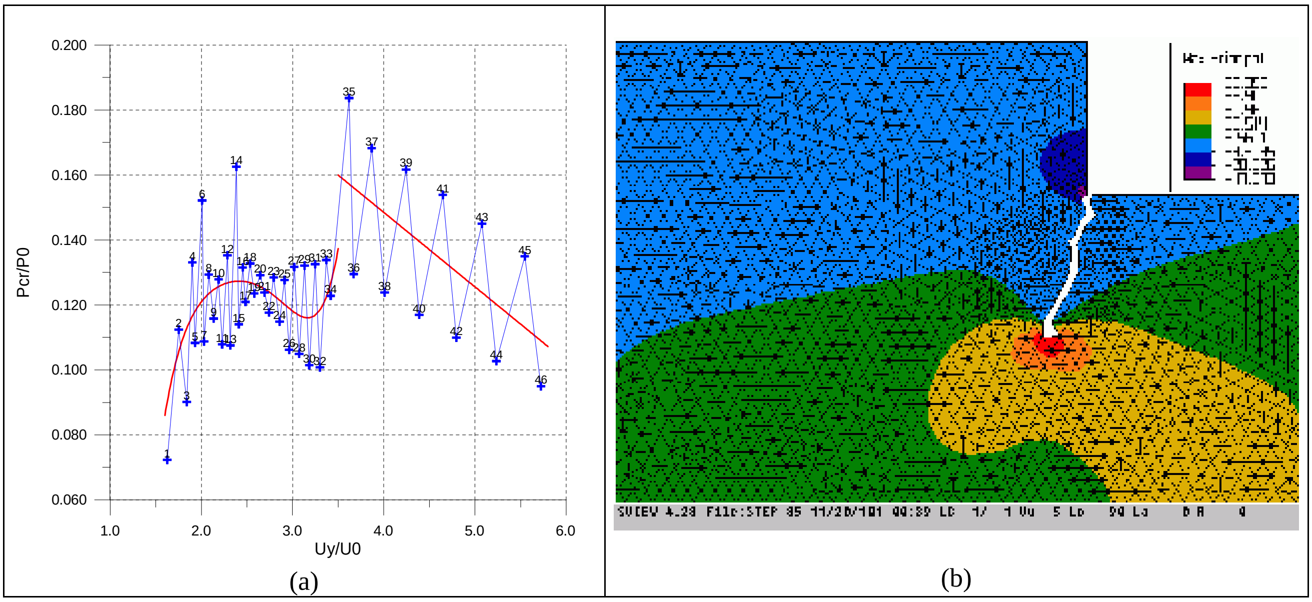


Fig. 8. Case #3 - Drucker-Prager criterion, model *“A”*, mesh with 2079 nodes.

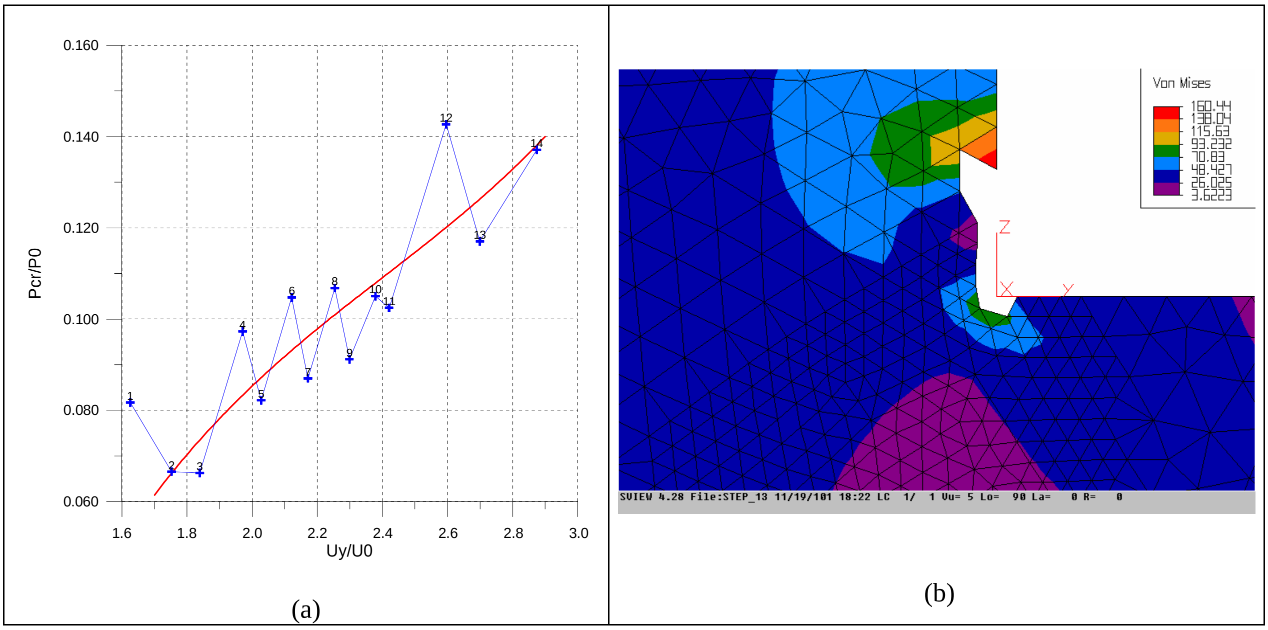


Fig. 9. Case #4 - Huber-Mises criterion, model *"A"*, mesh with 2079 nodes.

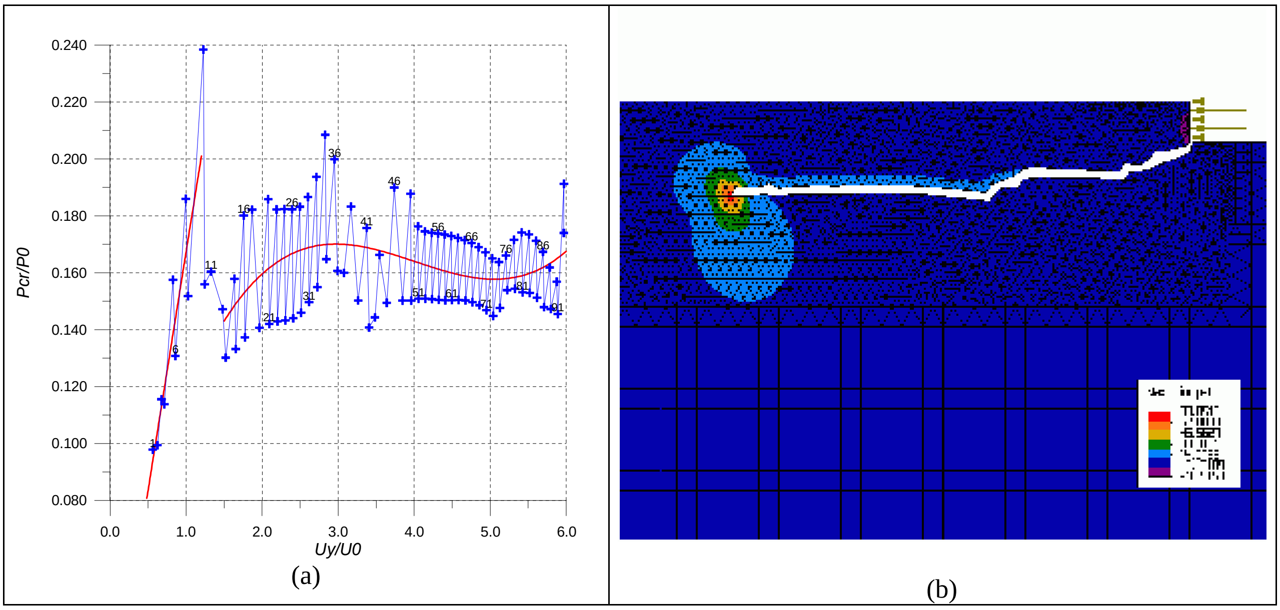
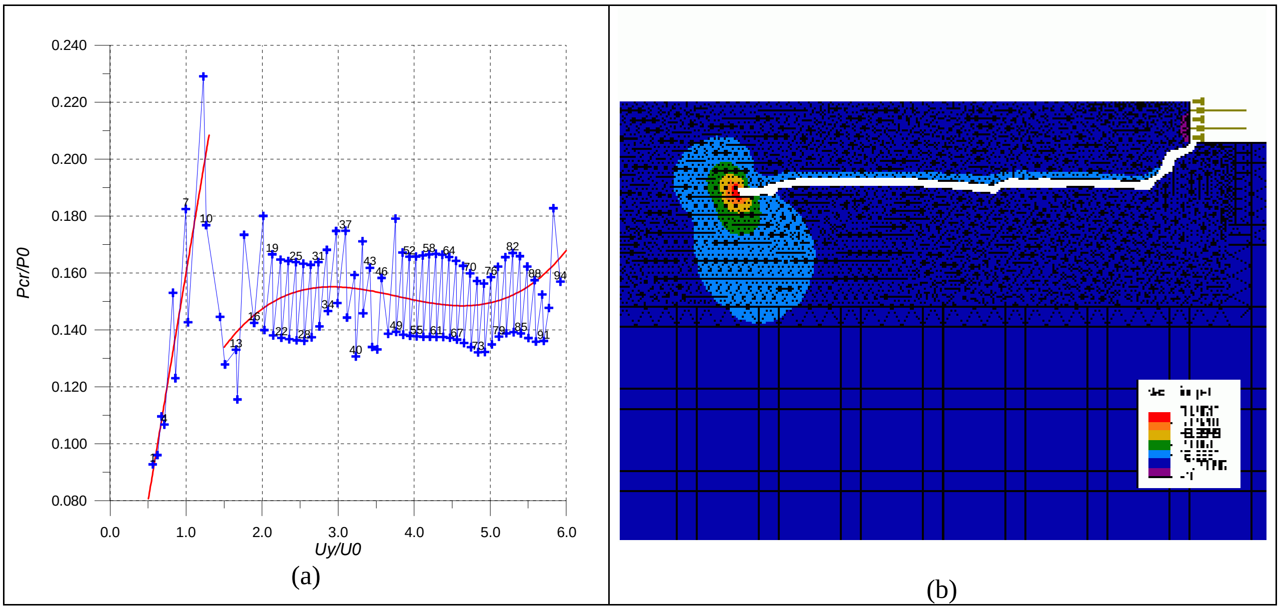


Fig. 10. Case #5 - ***PJ*** criterion, model ***"B"***, mesh with 3002 nodes.



(a) (b)

Fig. 11. Case #6 - Drucker-Prager criterion, model *"B"*, mesh with 3002 nodes.

Comparing presented results we can point out that shape of the crack and value of critical forces observed in case of ***JP*** and Drucker-Prager criterion are similar. This is result of comparable evaluation of the critical loading in the region of shear stresses (compression-tension values of principal stresses, Fig. 1). For Drucker-Prager criterion somewhat lower values of critical forces was evaluated (cf. Fig 7-8 and Fig 10-11). Huber-Mises criterion gives completely different results in crack shape and critical force. This is result of exceeding limit stresses in compression region where both mentioned above criteria give large reserve in the limited stresses.

Values of critical forces calculated by this method are visibly dependent on finite elements mesh density. For fine mesh slightly lower results can be observed. On the border between fine and coarse mesh domain a "force jump" occurs (cf. step 35 on Fig. 7a-b and Fig. 8a-b).

5. Conclusions

Presented analysis points out significant dependency of crack shape and its direction on kind of failure criterion used. Similar results were observed by other authors [5] in the shear test simulation by FEM with Burzyński (analogical to Drucker-Prager criterion) and Huber-Mises criteria.

Problems of crack propagation in brittle materials, in which biaxial stress state are dominating, can be analyzed with sufficient precision by Drucker-Prager criterion in the compression-tension region. In other regions in which principal stresses are both positive or negative, this simplification cannot be used.

In the regions of high pressure or triaxial stress state, differences observed in crack shape and critical force values between two-invariant dependent criteria (like Drucker-Prager or Burzynski) and three invariant (like Coulomb-Mohr, Ottosen, Lade or ***PJ*** criterion), are very significant. Some simple criteria, like Huber-Mises or Tresca, cannot be used for this media in any region.

Values of the critical forces calculated by "dead element" method are significantly dependent on the finite element mesh density.

6. References

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Abstract

Influence of three different types of the failure conditions on the shape and direction of the crack propagation in then elastic-brittle material is presented. Finite Elements Method and "death element" procedure have been used to modelling and analysis of the crack propagation.

Huber-Mises, Drucker-Prager and author (PJ) failure criterion [1] are applied to the concrete-like or rock-like materials.