

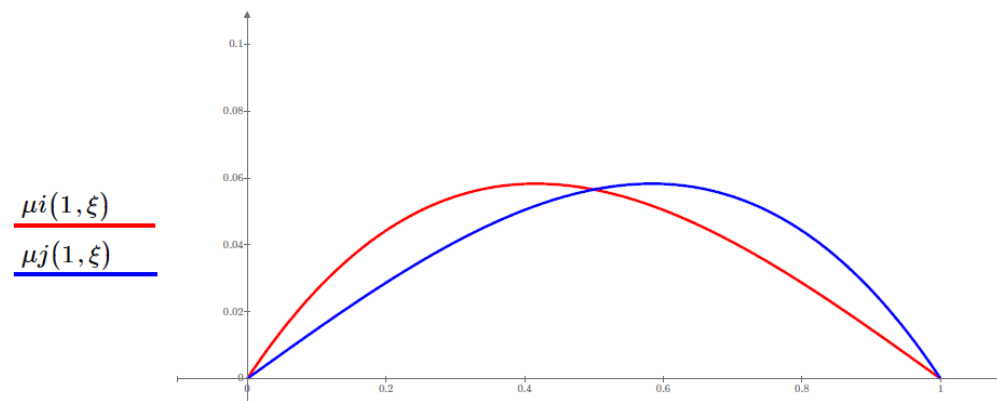
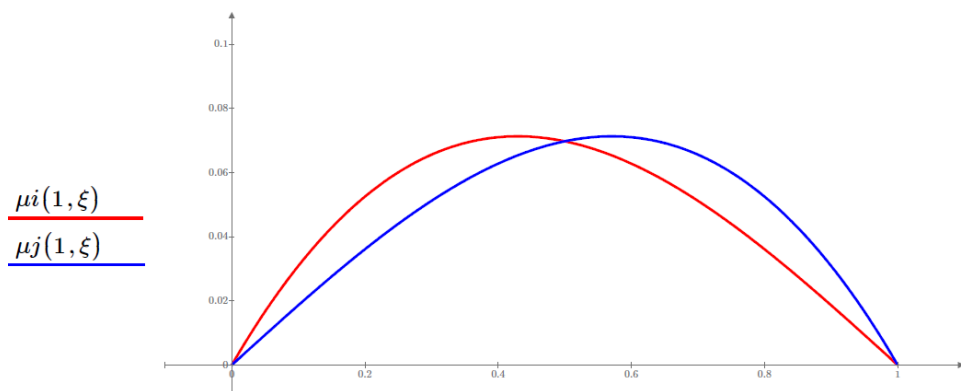
$$y(x) = A + B\xi + C \sin(\alpha\xi) + D \cos(\alpha\xi) \qquad \alpha = l \sqrt{\frac{S}{EJ}} \quad l\xi = x$$

$$\mu i(\alpha, \xi) := \frac{\sin(\alpha \cdot (1-\xi))}{\sin(\alpha)} - 1 + \xi$$

$$\mu j(\alpha, \xi) := \frac{\sin(\alpha \cdot \xi)}{\sin(\alpha)} - \xi$$

$$\mu i(\alpha, \xi) := \frac{-\sinh(\alpha \cdot (1-\xi))}{\sinh(\alpha)} + 1 - \xi$$

$$\mu j(\alpha, \xi) := \frac{-\sinh(\alpha \cdot \xi)}{\sinh(\alpha)} + \xi$$



$$\begin{aligned}
 M_i &= \frac{EJ}{l} \left[c(\alpha) \varphi_i + s(\alpha) \varphi_j - r(\alpha) \frac{u_j - u_i}{l} \right] & T_i &= -\frac{EJ}{l^2} \left[r(\alpha) \varphi_i + r(\alpha) \varphi_j - b(\alpha) \frac{u_j - u_i}{l} \right] \\
 M_j &= \frac{EJ}{l} \left[s(\alpha) \varphi_i + c(\alpha) \varphi_j - r(\alpha) \frac{u_j - u_i}{l} \right] & T_j &= \frac{EJ}{l^2} \left[r(\alpha) \varphi_i + r(\alpha) \varphi_j - b(\alpha) \frac{u_j - u_i}{l} \right] \\
 c(\alpha) &= \frac{\alpha}{d(\alpha)} (\sin \alpha - \alpha \cos \alpha) & d(\alpha) &= 2(1 - \cos \alpha) - \alpha \sin \alpha \\
 s(\alpha) &= \frac{\alpha}{d(\alpha)} (\alpha - \sin \alpha) & r(\alpha) &= s(\alpha) + c(\alpha) = \frac{\alpha^2}{d(\alpha)} (1 - \cos \alpha) & b(\alpha) &= \frac{\alpha^3}{d(\alpha)} \sin \alpha
 \end{aligned}$$

$$\begin{aligned}
 M_i &= \frac{EJ}{l} \left[c^I(\alpha) \varphi_i - c^I(\alpha) \frac{u_j - u_i}{l} \right] & T_i &= -\frac{EJ}{l^2} \left[c^I(\alpha) \varphi_i - r^I(\alpha) \frac{u_j - u_i}{l} \right] \\
 M_j &= 0 & T_j &= \frac{EJ}{l^2} \left[c^I(\alpha) \varphi_i - r^I(\alpha) \frac{u_j - u_i}{l} \right] \\
 c^I(\alpha) &= \alpha^2 \frac{\sin \alpha}{\sin \alpha - \alpha \cos \alpha} & r^I(\alpha) &= \alpha^3 \frac{\cos \alpha}{\sin \alpha - \alpha \cos \alpha}
 \end{aligned}$$

$$K(\sigma) = \frac{EJ}{l^2} \begin{array}{|c|c|c|c|} \hline \frac{\delta}{l} & \vartheta & -\frac{\delta}{l} & \vartheta \\ \hline \vartheta & \alpha l & -\vartheta & \beta l \\ \hline -\frac{\delta}{l} & -\vartheta & \frac{\delta}{l} & -\vartheta \\ \hline \vartheta & \beta l & -\vartheta & \alpha l \\ \hline \end{array},$$

$$\text{gdzie : } \alpha = \alpha(\sigma) = \frac{\sigma}{\Delta(\sigma)}(\sin \sigma - \sigma \cos \sigma),$$

$$\beta = \beta(\sigma) = \frac{\sigma}{\Delta(\sigma)}(\sigma - \sin \sigma),$$

$$\vartheta = \vartheta(\sigma) = \frac{\sigma^2}{\Delta(\sigma)}(1 - \cos \sigma),$$

$$\delta = \delta(\sigma) = \frac{\sigma^3}{\Delta(\sigma)} \sin \sigma,$$

$$\Delta(\sigma) = 2(1 - \cos \sigma) - \sigma \sin \sigma.$$

$$\sigma = l \sqrt{\frac{S}{EJ}}$$

$$\sin \sigma = \sigma - \frac{\sigma^3}{3!} + \frac{\sigma^5}{5!} - \frac{\sigma^7}{7!} + \dots,$$

$$\cos \sigma = 1 - \frac{\sigma^2}{2!} + \frac{\sigma^4}{4!} - \frac{\sigma^6}{6!} + \dots$$

$$\begin{aligned}\Delta(\sigma) &= 2 \left(1 - 1 + \frac{\sigma^2}{2!} - \frac{\sigma^4}{4!} + \frac{\sigma^6}{6!} - \dots \right) - \sigma \left(\sigma - \frac{\sigma^3}{3!} + \frac{\sigma^5}{5!} - \frac{\sigma^7}{7!} + \dots \right) = \\ &= \sigma^4 \left(\frac{1}{12} - \frac{\sigma^2}{180} + \frac{\sigma^4}{6720} - \frac{\sigma^6}{453600} + \dots \right),\end{aligned}$$

$$\begin{aligned}\alpha(\sigma) &= \frac{\sigma}{\Delta(\sigma)} \left(\sigma - \frac{\sigma^3}{3!} + \frac{\sigma^5}{5!} - \frac{\sigma^7}{7!} + \dots - \sigma + \frac{\sigma^3}{2!} - \frac{\sigma^5}{4!} + \frac{\sigma^7}{6!} - \dots \right) = \\ &= \frac{\frac{1}{3} - \frac{\sigma^2}{30} + \frac{\sigma^4}{840} - \frac{\sigma^6}{45360} + \dots}{\frac{1}{12} - \frac{\sigma^2}{180} + \frac{\sigma^4}{6720} - \frac{\sigma^6}{453600} + \dots}.\end{aligned}$$

Po wykonaniu dzielenia uzyskujemy wyrażenie:

$$\alpha(\sigma) = 4 - \frac{2}{15}\sigma^2 - \frac{11}{6300}\sigma^4 - \frac{1}{27000}\sigma^6 + \dots. \quad (6-134)$$

W analogiczny sposób obliczamy:

$$\beta(\sigma) = 2 + \frac{1}{30}\sigma^2 + \frac{13}{12600}\sigma^4 + \frac{11}{378000}\sigma^6 + \dots,$$

$$\vartheta(\sigma) = 6 - \frac{1}{10}\sigma^2 - \frac{1}{1400}\sigma^4 - \frac{1}{126000}\sigma^6 - \dots, \quad (6-135)$$

$$\delta(\sigma) = 12 - \frac{6}{5}\sigma^2 - \frac{1}{700}\sigma^4 - \frac{1}{63000}\sigma^6 - \dots.$$

$$\begin{aligned}
 K(\sigma) = & \frac{EJ}{l^2} \left\{ \begin{array}{|c|c|c|c|} \hline \frac{12}{l} & 6 & -\frac{12}{l} & 6 \\ \hline 6 & 4l & -6 & 2l \\ \hline -\frac{12}{l} & -6 & \frac{12}{l} & -6 \\ \hline 6 & 2l & -6 & 4l \\ \hline \end{array} \right. - \frac{\sigma^2}{30} \begin{array}{|c|c|c|c|} \hline \frac{36}{l} & 3 & -\frac{36}{l} & 3 \\ \hline 3 & 4l & -3 & -l \\ \hline -\frac{36}{l} & -3 & \frac{36}{l} & -3 \\ \hline 3 & -l & -3 & 4l \\ \hline \end{array} - \\
 & - \frac{\sigma^4}{12600} \begin{array}{|c|c|c|c|} \hline \frac{18}{l} & 9 & -\frac{18}{l} & 9 \\ \hline 9 & 22l & -9 & 13l \\ \hline -\frac{18}{l} & -9 & \frac{18}{l} & -9 \\ \hline 9 & 13l & -9 & 22l \\ \hline \end{array} - \frac{\sigma^6}{378000} \begin{array}{|c|c|c|c|} \hline \frac{6}{l} & 3 & -\frac{6}{l} & 3 \\ \hline 3 & 14l & -3 & -11l \\ \hline -\frac{6}{l} & -3 & \frac{6}{l} & -3 \\ \hline 3 & -11l & -3 & 14l \\ \hline \end{array} \dots \left. \vphantom{\frac{EJ}{l^2}} \right\} (6-136)
 \end{aligned}$$