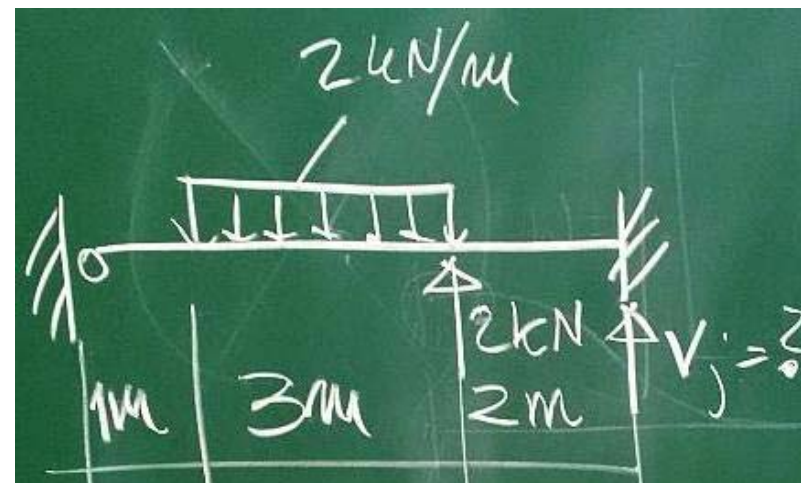
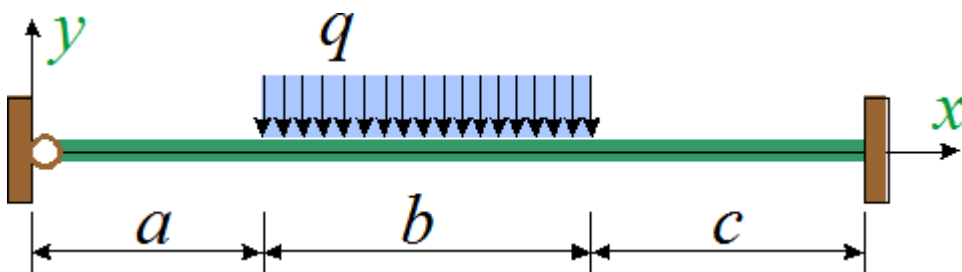


Należy obliczyć siłę węzłową  $V_j$  wykorzystując zasadę prac wirtualnych i funkcję kształtu elementu belkowego

$$q := 2 \cdot \frac{\text{kN}}{\text{m}} \quad P := 2 \text{ kN} \quad a := 1 \cdot \text{m} \quad b := 3 \cdot \text{m} \quad c := 2 \cdot \text{m} \quad L := a + b + c$$

$$\xi_1 := \frac{a}{L} = 0.166667 \quad \xi_2 := \frac{a+b}{L} = 0.666667 \quad \xi_P := \xi_2$$



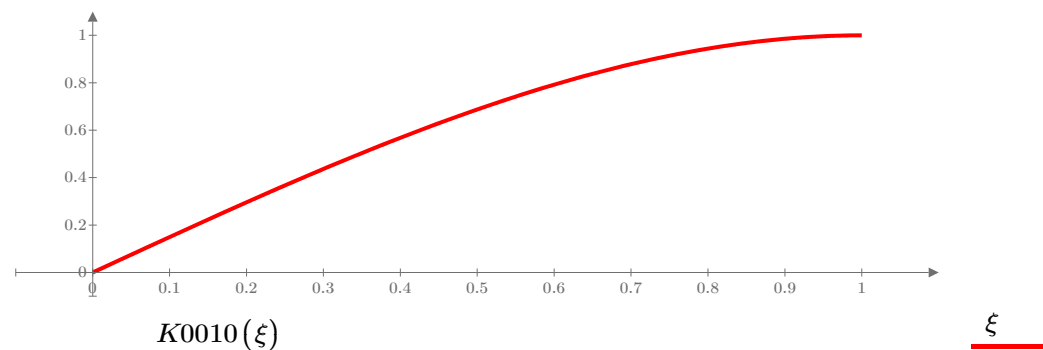
$$K0010(\xi) := \frac{\xi}{2} \cdot (3 - \xi^2)$$

Równanie pracy wirtualnej

$$V_j \cdot \delta + P \cdot \delta \cdot K0010(\xi_P) - \int_{x_1}^{x_2} q \cdot \delta \cdot K0010(\xi) dx = 0$$

$$\delta \cdot \left( V_j \cdot 1 + P \cdot K0010(\xi_P) - q \cdot L \cdot \int_{\xi_1}^{\xi_2} K0010(\xi) d\xi \right) = 0$$

$$V_j := q \cdot L \cdot \int_{\xi_1}^{\xi_2} K0010(\xi) d\xi - P \cdot K0010(\xi_P) = 1.7512 \text{ kN}$$



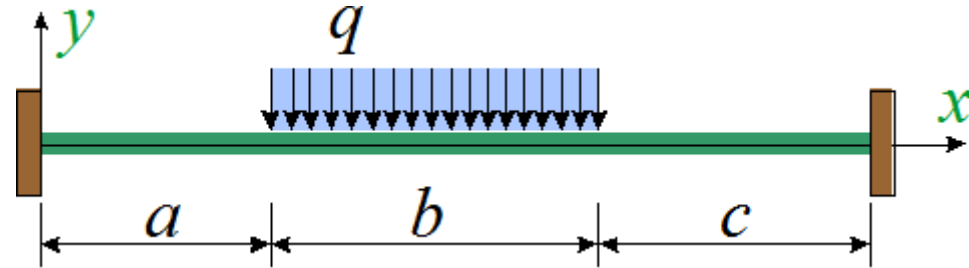
## Definicja wielomianów Hermite'a dla belki obustronnie sztywno zamocowanej

$$H1000(\xi) := 1 - 3 \cdot \xi^2 + 2 \cdot \xi^3$$

$$H0100(\xi) := \xi \cdot (1 - 2 \cdot \xi + \xi^2)$$

$$H0010(\xi) := \xi^2 \cdot (3 - 2 \cdot \xi)$$

$$H0001(\xi) := -\xi^2 \cdot (1 - \xi)$$



Wyprowadzenie wzoru  $H1000(\xi)$

Warunki brzegowe:

$$y(0) = \delta \quad y'(0) = 0$$

$$y(l) = 0 \quad y'(l) = 0$$

$$y(x) = A + B \cdot x + C \cdot x^2 + D \cdot x^3$$

$$y'(x) = B + 2 C \cdot x + 3 D \cdot x^2$$

$$y(0) = A = \delta \quad y'(0) = B = 0$$

$$y(l) = \delta + C \cdot l^2 + D \cdot l^3 = 0$$

$$y'(l) = 2 C \cdot l + 3 D \cdot l^2 = 0$$

$$\left. \begin{aligned} C \cdot l + D \cdot l^2 &= -\frac{\delta}{l} \\ C \cdot l + \frac{3}{2} D \cdot l^2 &= 0 \end{aligned} \right\}$$

$$\frac{1}{2} D \cdot l^2 = \frac{\delta}{l}$$

$$D = \frac{2 \delta}{l^3}$$

$$C = \frac{-3 \delta}{l^2}$$

$$y(x) = \delta \cdot \left( 1 - 3 \cdot \frac{x^2}{l^2} + 2 \cdot \frac{x^3}{l^3} \right)$$

$$y(\xi) = \delta \cdot (1 - 3 \cdot \xi^2 + 2 \cdot \xi^3)$$

$$y(\xi) = \delta \cdot H1000(\xi)$$

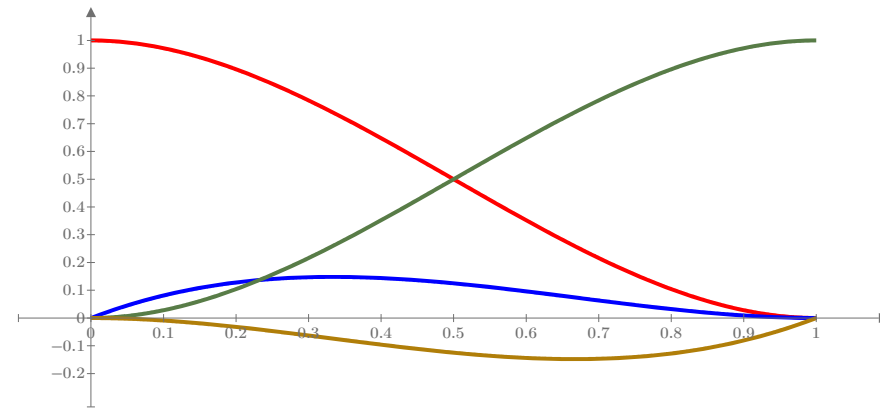
$$H1000(\xi) = (1 - 3 \cdot \xi^2 + 2 \cdot \xi^3)$$

$$H1000(\xi)$$

$$H0100(\xi)$$

$$H0010(\xi)$$

$$H0001(\xi)$$



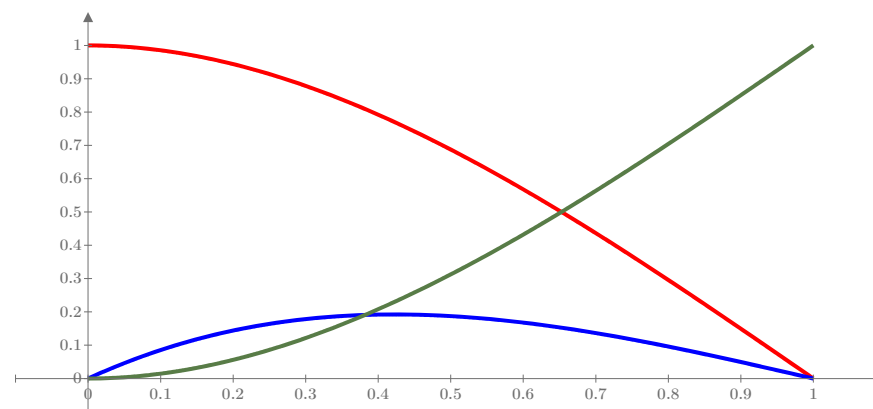
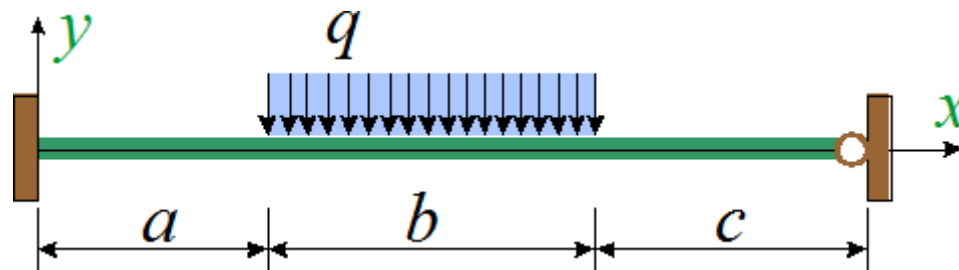
$\xi$

## Definicja wielomianów Hermite'a dla belki połączonej przegubowo z prawym węzłem - $G(\xi)$

$$G_{1000}(\xi) := 1 - \frac{3}{2} \cdot \xi^2 + \frac{1}{2} \cdot \xi^3$$

$$G_{0100}(\xi) := \frac{\xi}{2} \cdot (2 - 3 \cdot \xi + \xi^2)$$

$$G_{0010}(\xi) := \frac{\xi^2}{2} \cdot (3 - \xi)$$



Wyprowadzenie wzoru  $G_{0100}(\xi)$

Warunki brzegowe:

$$y(0) = 0 \quad y'(0) = \varphi$$

$$y(l) = 0 \quad y''(l) = 0$$

$$y(x) = A + B \cdot x + C \cdot x^2 + D \cdot x^3$$

$$y'(x) = B + 2 C \cdot x + 3 D \cdot x^2$$

$$y''(x) = 2 C + 6 D \cdot x$$

$$y(0) = A = 0 \quad y'(0) = B = \varphi$$

$$y(l) = \varphi \cdot l + C \cdot l^2 + D \cdot l^3 = 0$$

$$y''(l) = 2 C + 6 D \cdot l = 0$$

$$\left. \begin{array}{l} C + D \cdot l = -\frac{\varphi}{l} \\ C + 3 D \cdot l = 0 \end{array} \right\} \rightarrow 2 D \cdot l = \frac{\varphi}{l^2} \rightarrow D = \frac{\varphi}{2 l^2} \rightarrow C = \frac{-3 \varphi}{2 l}$$

$$y(x) = \varphi \cdot \left( x - \frac{3}{2} \cdot \frac{x^2}{l} + \frac{x^3}{2 l^2} \right)$$

$$\rightarrow y(\xi) = \frac{\varphi \cdot l}{2} (2 \xi - 3 \cdot \xi^2 + \xi^3)$$

$$\rightarrow y(\xi) = \varphi \cdot l G_{0100}(\xi)$$

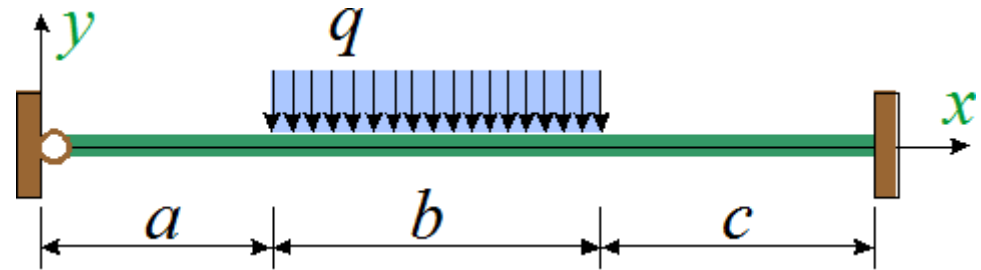
$$\rightarrow G_{0100}(\xi) = \frac{\xi}{2} \cdot (2 - 3 \cdot \xi + \xi^2)$$

## Definicja wielomianów Hermite'a dla belki połączonej przegubowo z lewym węzłem - $K(\xi)$

$$K1000(\xi) := 1 - \frac{3}{2} \cdot \xi + \frac{1}{2} \cdot \xi^3$$

$$K0010(\xi) := \frac{\xi}{2} \cdot (3 - \xi^2)$$

$$K0001(\xi) := \frac{\xi}{2} \cdot (\xi^2 - 1)$$



Wyprowadzenie wzoru  $K1000(\xi)$

Warunki brzegowe:

$$y(0) = \delta \quad y''(0) = 0$$

$$y(l) = 0 \quad y'(l) = 0$$

$$y(x) = A + B \cdot x + C \cdot x^2 + D \cdot x^3$$

$$y'(x) = B + 2 C \cdot x + 3 D \cdot x^2$$

$$y''(x) = 2 C + 6 D \cdot x$$

$$y(0) = A = \delta \quad y''(0) = 2 C = 0$$

$$y(l) = \delta + B \cdot l + D \cdot l^3 = 0$$

$$y'(l) = B + 3 D \cdot l^2 = 0$$

$$\left. \begin{array}{l} B + D \cdot l^2 = -\frac{\delta}{l} \\ B + 3 D \cdot l^2 = 0 \end{array} \right\} \rightarrow$$

$$2 D \cdot l^2 = \frac{\delta}{l}$$

$$D = \frac{\delta}{2 l^3}$$

$$B = \frac{-3 \delta}{2 l}$$

$$y(x) = \delta \cdot \left( 1 - \frac{3}{2} \cdot \frac{x}{l} + \frac{x^3}{2 l^3} \right)$$

$$\rightarrow y(\xi) = \delta \left( 1 - \frac{3}{2} \cdot \xi + \frac{1}{2} \xi^3 \right)$$

$$\rightarrow y(\xi) = \delta K1000(\xi)$$

$$\rightarrow K1000(\xi) = 1 - \frac{3}{2} \cdot \xi + \frac{1}{2} \xi^3$$

$$\begin{array}{l} K1000(\xi) \\ K0010(\xi) \\ K0001(\xi) \end{array}$$

